SYLVAN RESEARCH INSTITUTE

Research in Action:

Mathematics

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Mathematics

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The staff of Rockman et al includes researchers with advanced degrees in education, cognitive science, communications research, child development, research design, educational technology, psychology, and the humanities. Since 1990, Rockman et al has conducted hundreds of evaluations and research studies and has often served as the external evaluator for grant-funded projects supported by foundations, state and federal agencies, and private industry.

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INTRODUCTION

Sylvan Learning offers a variety of programs to help kindergarten through high school students meet the challenges of a rigorous mathematics curriculum. Among the programs designed to remediate gaps in students' skills and provide enrichment are the Ace it! small-group instructional program, mathematics camps, and Sylvan's new digital teaching platform, SylvanSync. This highly personalized program uses tablet computers to deliver and manage digital content, which teachers then use to tailor instruction to students' needs and make adjustments based on their progress and learning arcs. All of Sylvan's programs can be delivered at Sylvan Learning Centers or at schools or community sites.

Sylvan Learning maximizes the instructional effectiveness of each student's program by:

- Creating an instructional plan for each student;
- Using a curriculum that is aligned to national and state education standards;
- Providing ongoing evaluation and daily monitoring to track achievement;
- Motivating students by rewarding effort and achievement;
- Involving parents in their child's educational program; and
- Involving classroom teachers, when appropriate, in students' educational programs.

Like all of Sylvan's instructional offerings, these mathematics programs are based on widely accepted theories of teaching and learning and recent research in the field. This report reviews the research behind the features and designs of Sylvan's mathematics programs, discussing theoretical views on instruction and proficiency as well as the standards and reform efforts undertaken by the National Research Council (NRC), the National Council of Teachers of Mathematics (NCTM), the National Mathematics Advisory Panel (NMAP), and, most recently, the National Governors' Association (NGA) and Council of Chief State School Officers (CCSSO), developers of the new Common Core State Standards for Mathematics (CCSSM).

The focus of much of the research, and a primary impetus of the new Common Core standards, is growing concern about the widening gap between students' academic knowledge and skills at the end of K–12 schooling and the skills that 21st-century college, careers, and the global economy require. A National Assessment of Educational Progress (NAEP) test administered in the early 1990s indicated that about 13–16% of 12th-grade students were proficient in mathematics (Mullis et al., 1994). Data from a more recent U.S. administration of the ACT's College Readiness Test in Mathematics revealed that only one-third to one-half are college and career ready (ACT, 2010); reports of the numbers of remedial courses taken by college freshmen seem to bear those figures out (Conley, 2012). Results from the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA), conducted every four and three years, respectively, to compare achievement across nations, also suggest that U.S. students do not perform as well as many of their peers around the world (Layton, 2013).

These findings have provoked laments from U.S. researchers, journalists, and policymakers about the state of mathematics instruction. But they have also inspired researchers, policymakers, and practitioners to rethink standards and instruction for students, teacher preparation and professional development, and the use of assessment data, especially formative data that can inform instructional decisions (Gonzales et al., 2008; Wiliam, 2011).

Sylvan Learning programs reflect the recent changes in standards and reform efforts, along with other related areas of research, including the findings on effective out-of-school programs (Beckett et al., 2009; Miller, Snow, & Lauer, 2004; Pashler & Bain, 2007; Rockman & Fontana, 2009; Strobel, 2008). That research suggests that participation in after-school programs can improve more than students' academic performance: It can also build self-confidence, self-esteem, and positive attitudes toward school (Durlak & Weissberg, 2007). Sylvan Learning's programs always attended to student motivation. However, based on this new research, and a growing number of studies on the importance of motivation in helping students develop an "academic mindset" (Farrington et al., 2012), Sylvan Learning is now tracking attitudinal changes using the Student Outlook Survey developed by Rockman et al.

Drawing on studies of effective out-of-school learning, Sylvan programs encourage student success by providing:

- Experienced, highly trained **staff** who know how to work with children with diverse learning preferences and those who thrive with a more individualized approach.
- Quality **curriculum** that is (a) aligned to school curriculum and to local, state, and national standards; (b) age- and grade-level appropriate; and (c) delivered with effective **instructional techniques**, including varied pedagogical styles, personalized instruction, and engaging, interactive learning experiences.
- Programs that provide adequate **structure** but also **flexibility** in session length and program duration.
- Strong and positive **partnerships** with classroom instructors, parents, and schools.
- Quality **resources**, including technology and facilities that foster sustained involvement in a safe, healthy environment.
- Well-aligned **evaluation** and **research** components to provide feedback on the programs.

With the launch of SylvanSync, Sylvan has also paid particular attention to the research on adaptive learning and learning progressions, as ways not only to chart each student's learning path but also take full advantage of the potential of digital technology. The report reviews studies of that potential, and research on technology's role in bolstering the effect of after-school programs in improving students' self-confidence and attitudes toward school (Collaborative for Academic, Social, and Emotional Learning, 2007).

SYLVAN MATHEMATICS PROGRAMS

Sylvan offers a variety of mathematics programs, each with unique features. **SylvanSync Mathematics**, which uses an integrated technology platform, is Sylvan's most individualized program. The digital resources adapt based on students' performance, helping teachers motivate students and provide scaffolded, highly individualized instruction. SylvanSync Mathematics provides instruction to students in levels K–9 in numbers and operations, geometry and measurement, algebra, and data analysis, statistics, and probability skills from the most basic to more sophisticated modeling and problem solving skills. Students also have many opportunities to practice basic calculations, apply algorithms, and solve problems.

The **Ace It! Math** curriculum is a customized, proprietary small-group math curriculum developed by Sylvan. The curriculum is based on the NCTM and NMAP curricular focal points for pre-kindergarten through grade 8 mathematics (NCTM, 2005). It includes both content standards (numbers and operations, algebra, geometry, measurement, data analysis, and probability) and process standards (problem solving, reason and proof, communication, connections, and representations). The math program is built on these foundations and uses a balanced approach for building the student's overall mathematics skills. Designed for groups of 8–10 students, Ace it! Math provides opportunities for collaboration and communication throughout the program.

The **Sylvan Mathematics camps** are small-group programs that focus on math skills as well as problem-solving development. These camps are designed to enrich a student's academic experience through engaging, creative, and collaborative activities.

THEORETICAL VIEWS ON MATHEMATICS LEARNING

The CCSSM reflect a half-century of research, over two decades of reform, and general consensus that mathematics is more than a set of computational skills (Battista, 1999). To develop powerful mathematical thinking, most researchers and educators now agree that instruction must include both content and process skills, and encourage students to invent, test, and refine their own ideas rather than merely follow procedures given to them by others. This constructivist view reflects developmental theories set forth by Jean Piaget (1952) and more recently by scientists attempting to connect brain function to psychology (Crick, 1994).

Sylvan's approach integrates both a cognitivist and constructivist view of learning. A cognitivist approach is evident in Sylvan's view that the learner is an active participant in acquiring knowledge, and that knowledge is organized into schemata that support student learning. Sylvan's instructional materials also include demonstrations, manipulatives, and examples that build mental models for the learner. This sample lesson in this link shows the use of graphic representations to help students understand fractional parts of a region. Centers are equipped with similar fraction manipulatives that can be used along with this lesson to provide hands-on experiences for students.

Sylvan's individualized approach supports the constructivist theory that each learner creates meaning from experience. Follow this link to see a sample lesson that demonstrates how students can create meaning through mathematical investigation and draw conclusions from a data set.

Students also have opportunities to reflect on their prior experiences to make meaning of new ones. Among these opportunities are the Learning Log prompts that students respond to at the end of each tutoring session, reflecting on their knowledge, understanding, and learning experiences. See an example of the Learning Log prompt on page 28.

Acquiring Mathematical Proficiency

In a report commissioned by the National Research Council (NRC, 2001), the Committee on Mathematics Learning summarized research in the field, using the term "mathematical proficiency" to refer to expertise, competence, knowledge, and facility in mathematics.

The committee identified five strands of proficiency, included in the National Research Council report, *Adding It Up* (NRC, 2001).

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy

Carpenter, Franke, and Levi (2003) also propose four forms of mental activity related to developing proficiency in mathematics: constructing relationships; extending and applying mathematical and scientific knowledge; justifying and explaining generalizations and procedures; and developing a sense of identity that includes taking responsibility for making sense of mathematical and scientific knowledge.

Pre-K Proficiency

The Committee on Mathematics Learning report (NRC, 2001) noted that many math educators and researchers believe that young children have a basic, almost intuitive level of math proficiency, and by the time they are ready for kindergarten can solve simple problems: They can count stars, divide marbles, figure out how far it is to walk somewhere, and notice that a sibling has more candies than they do. Most children, according to the report, also seem to enjoy using mathematics to solve problems (NRC, 2001).

However, when students move beyond what they understand intuitively, their comfort level begins to falter. By the time they leave school, many are apprehensive and unsure about performing any except the most trivial math tasks (Battista, 1999). Much recent research explores how this natural proficiency can be strengthened as children advance in school and encounter more complex mathematical problems and procedures.

From Elementary to Secondary Grades

Children begin to solve single-digit problems in the early grades using intuitive and tangible methods, but as math becomes more sophisticated, they have to choose among different procedures. Counting becomes abbreviated and rapid, and students begin to use arithmetic properties to simplify computations. Multiplication and division are more difficult because students need specific pattern-based knowledge, orchestrated into procedures that they must carry out quickly in order to be successful. Learning to use algorithms-procedures that can be executed in the same way to solve a variety of problems-for computation with multi-digit numbers is also an important and challenging part of developing mathematical proficiency. A variety of instructional approaches are effective in helping students learn multi-digit arithmetic, focusing on the base-ten structure and encouraging students to use and build on algorithms that they already understand.

In grades pre–K through 8, rational numbers, represented as fractions and decimals, present a challenge because of the many properties that students must learn. Although early notions of partitioning, sharing, and measuring provide a starting point, students often have difficulty with fractions, which can present "an obstacle to further progress in mathematics and other domains dependent on mathematics, including algebra" (NMAP, 2008, p. 28).

Foundations for Algebra

Much attention has been given to the teaching of algebra as a benchmark for mathematical proficiency, in part because so many students have difficulty transitioning from arithmetic to algebra, which is based on symbolism, equation solving, and emphasis on relationships among quantities. “Algebra for all” has become a mantra in the mathematics education field and a focus for researchers, policymakers, and practitioners, including the National Mathematics Advisory Panel (2008, p. xv). Kaput (2000) advocates “algebrafying” the entire K–12 curriculum, arguing that fulfilling the promise of algebra for all can eliminate “the most pernicious curricular element of today’s school mathematics-late, abrupt, isolated, and superficial high school algebra courses” (pp. 1–24). Carpenter and colleagues (2003), who have been working on the Early Algebra Project for the past decade, believe that teachers should engage children in learning the general principles of mathematics, including algebra, as they learn arithmetic. They assert that learning arithmetic in isolation deprives students of powerful ways of thinking about mathematics, and that students studying high school algebra do not see the procedures that they use to solve equations and simplify expressions as similar to the procedures learned for arithmetic.

Secondary Mathematics

The focus on deficiencies in mathematics teaching and learning has been on secondary classrooms in particular, prompted in part by the TIMSS findings (Silver, 1998), which showed that, at grades 7 and 12, U.S. students performed poorly in mathematics compared to students in much of the rest of the world; eighth-graders fared somewhat better, with average performance in algebra, fractions, data representation, analysis, and probability, but below-average scores in geometry, measurement, and proportionality. Findings regarding curricula indicated that the U.S. school mathematics curriculum is unfocused, repetitive, and not sufficiently demanding, and that grade 8 instruction is oriented neither toward understanding nor toward intellectual challenge (Silver, 1998).

STANDARDS AND REFORM EFFORTS TO BUILD MATHEMATICS PROFICIENCY

At least a decade before the troublesome reports of how U.S. students compared to their peers around the world, and the subsequent calls for a national effort to reform mathematics curriculum and instruction, the U.S. mathematics education community had already begun to study and develop an improved mathematics curriculum, especially for secondary students. In 1989, NCTM issued its *Curriculum and Evaluation Standards for School Mathematics*, which provided a vision of mathematics education and served as a catalyst for a national standards movement. These first national standards were inspired by research but did not reflect empirical testing of curricula. In 2000, NCTM issued its *Principles and Standards for School Mathematics (PSSM)*, a statement designed to reflect the previous ten years of research on diverse models of curricular change and enriched discourse about mathematics curricula. This document lays out four principles of curricular design, including equity, or high expectations and strong support for all students; coherent curricula instead of disconnected activities; teacher professionalism, including knowledge of curricula and learning; and effective use of assessment and technology in the service of mathematics learning.

More recently, authors of the introductory chapter in *How Students Learn: History, Mathematics and Science in the Classroom* (Fuson, Kalchman, & Bransford, 2005) argue that there are three basic instructional principles vital to helping students become proficient in mathematics, which they also say are rarely in place in classrooms.

- Building on the strategies and mathematical reasoning approaches that students bring with them to school to connect formal and informal learning
- Equally emphasizing conceptual understanding and procedural fluency
- Emphasizing metacognitive approaches that enable student self-monitoring (p. 239)

Mathematics curriculum designers-including those at Sylvan Learning-have structured curricula around these principles.

Sylvan's approach to instruction integrates these three instructional principles as demonstrated by the following content:

Principle 1 emphasizes the importance of building on the strategies and mathematical reasoning, demonstrated in this lesson in which students bring prior knowledge of dividing a pizza into slices to understand and compare fractions.

Principle 2 emphasizes conceptual understanding and procedural fluency; in this lesson students develop both of these skills by comparing place value of decimals.

Principle 3 emphasizes metacognitive approaches. In this lesson students develop strategies to self-check solutions to equations.

The efforts of the NCTM and other researchers and educators have produced significant curricular changes, though not without controversy. When the 1989 standards were first released, opponents of reform worried that an emphasis on process over content would weaken the curriculum and lower students' proficiency. Because of the lag time involved in evaluating reform efforts, the jury was out until a decade later, when data on some of the first large-scale implementations of reformed curricula indicated that students did as well on skills as students in traditional curricula and better on understanding concepts and problem solving. Schoenfeld's (2002) research summary argued that standards-based reform is more likely to work best when implemented as part of a systemic effort in which curriculum, assessment, and professional development are aligned. Other findings also suggested that reform was going in the right direction. The authors of *Standards-based School Mathematics Curriculum: What Are They? What Do Students Learn?* maintain that not only do many more students do well, but "the racial performance gap diminishes substantially" (Senk & Thompson, 2003, p. 17).

Other evidence was not as positive. A RAND Mathematics Study Panel report commissioned by the U.S. Department of Education's Institute of Education Sciences (IES), *Mathematical Proficiency for All Students: Toward a Strategic Research and Development Program in Mathematics* (Ball, 2003), cited results of the National Assessment of Educational Progress showing less than adequate mathematical competence in students graduating from high school as well as persistent gaps between white students and students of color and between middle-class students and students living in poverty (Rand, 2003). The study confirmed agreement on the broad goals for mathematics proficiency and emphasized significant investments in improving mathematics on the part of the federal government and school systems, but still described the urgent need for improvement in the teaching and learning of mathematics. The 2007 TIMSS study showed that minimal improvements had been made after a decade of reform efforts, in spite of the massive changes in mathematics curricula (Gonzales et al., 2008).

The National Mathematics Advisory Panel (2008) made subsequent recommendations for curricula and instruction to help students develop mathematics proficiency and build the foundations for algebra, which provided an important conceptual framework for planning instruction to help students become mathematically proficient.

THE COMMON CORE STATE STANDARDS FOR MATHEMATICS

The Common Core State Standards (CCSS, 2010), released in June of 2010, were the result of a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers-and, more broadly, the result of reform efforts of the previous two decades. Individual teachers, along with teachers' organizations such as the National Education Association (NEA) and the American Federal of Teachers (AFT), provided feedback, and the National Council of Teachers of Mathematics, the National Council of Supervisors of Mathematics (NCSM), the Association of State Supervisors of Mathematics (ASSM), and the Association of Mathematics Teacher Educators (AMTE) strongly supported the new national standards.

> "The Common Core State Standards provide a consistent, clear understanding of what students are expected to learn, so teachers and parents know what they need to do to help them. The standards are designed to be robust and relevant to the real world, reflecting the knowledge and skills that our young people need for success in college and careers. With American students fully prepared for the future, our communities will be best positioned to compete successfully in the global economy." (CCSS, 2010)

Broadly speaking, the main difference between the CCSSM and previous efforts, as well as most state standards for mathematics, is that the new standards have fewer but more rigorous standards. Data from studies like TIMSS revealed that high-performing countries focus on fewer mathematics topics but with greater care and detail than the U. S. curriculum had previously done. Instead of a curriculum that is "a mile wide and an inch deep," the Common Core standards emphasize depth of understanding over breadth of material content. Along with their rigorous content, these standards are built on the existing strengths of current state standards and on documents such as *Principles and Standards of Mathematics* (NCTM, 2000) and *Curriculum Focal Points* (NCTM, 2006). They have also been internationally benchmarked so that students can succeed in a global economy and society.

To build a deep foundation of mathematical understanding with the same goals for all students, the CCSSM standards align curriculum, instruction, and assessment. The grade K–2 standards focus on addition and subtraction of whole numbers and the quantities they measure. The mathematics standards for grades 3–5 focus on multiplication and division along with facilitating fraction concepts. The standards for grades 6 and 7 focus on proportional reasoning and handling equations and expressions for a fuller sense of the number system; and grade 8 standards focus on the beginnings of algebra, mostly in terms of linear equations. High school standards focus on data and statistics, applying concepts to trigonometry and calculus with rich work in modeling and in multiple representations for social sciences such as economics or sociology. The sequence of this mathematical content is articulated developmentally and builds progressively. To nurture this in-depth mathematical foundation, the standards emphasize conceptual understanding and procedural skills equally. For example, fifth-graders should be able to conceptually demonstrate how fractions can be equivalent as well as to procedurally express those fractions in simplest form.

Standards for Mathematical Practice

The new Common Core Standards for Mathematical Practice include the processes that all students should understand and develop over the entire course of their schooling, as well as the strands of mathematical proficiency laid out in the NRC *Adding It Up* report (2001): adaptive reasoning, strategic competence, conceptual understanding of operations and relations, procedural fluency, and productive disposition-the sense that math is useful and that students can "do" math. The wording of the new process standards differs somewhat from the previous NCTM standards, but the skills are much the same. Table 1 provides a crosswalk between the two, along with language that students might use to describe the process.

Table 1. The NCTM Process Standards vs. the CCSS for Mathematical Practice

NCTM Process Standards	CCSS Mathematical Practice	Student-Friendly Language (White & Dauksas, 2012, p. 442)
Problem Solving	1. Make sense of problems and persevere in solving them.	"I can try many times to understand and solve a math problem."
Reasoning and Proof	2. Reason abstractly and quantitatively.	"I can think about the math problems in my head, first."
Communication	3. Construct viable arguments and critique the reasoning of others.	"I can make a plan, called a strategy, to solve the problem and discuss other students' strategies, too."
Representations	4. Model with mathematics.	"I can use math symbols and numbers to solve the problem."
Representations	5. Use appropriate tools strategically.	"I can use math tools, pictures, drawings, and objects to solve the problem."
Connections	6. Attend to precision.	"I can check to see if my strategy and calculations are correct."
Connections	7. Look for and make use of structure.	"I can use what I already know about math to solve the problem."
Reasoning and Proof	8. Look for and express regularity in repeated reasoning.	"I can use a strategy that I used to solve another math problem."

Making Sense of Problems and Persevering (Problem-Solving. Being able to solve problems involves using mathematics in varying ways as well as being able to apply mathematics learned from content strands. Students acquire new mathematics knowledge as they solve problems. They should be able to solve appropriate problems at all levels: Learning to use strategies such as guessing and checking, making tables and diagrams, looking for patterns, working backwards, and solving a simpler problem all foster good problem-solving skills. Problem solving not only gives students a way to integrate the strands of mathematical proficiency; it also gives teachers an opportunity to assess what students are doing and learning, including their strategic competence. Strategic competence is the ability to formulate mathematical problems, represent them, solve them, and explain solutions. Students should have a variety of solution strategies as well as an understanding of which strategies are useful for a particular problem situation. They should be able to tackle routine as well as non-routine problems for which they do not have an immediate solution strategy, and they should be able to persevere in trying different methods until they arrive at a solution, continually asking themselves, "Does this make sense?" (CCSSM, 2010).

The SylvanSync Mathematics program weaves problem solving through all lessons and provides explicit instruction for both the problem-solving process and problem-solving strategies.

Follow this link for a sample lesson that introduces the four-step problem-solving process woven through all content.

This lesson provides instruction in a variety of problem-solving strategies and provides practice with each strategy.

This lesson demonstrates drawing diagrams to solve geometry problems.

This lesson demonstrates the use of linear systems to solve real-world problems.

Reasoning Abstractly and Quantitatively (Reasoning and Proof). Being able to reason in mathematics involves informal explanation and justification as well as inductive and deductive reasoning. Students should be able to develop and evaluate mathematical conjectures and, in doing so, answer the question "Why does this work?" As they progress through grade levels, they should be able to develop systematic means of evaluation so that they can offer arguments and informal proofs. Elementary students should also be able to develop proof by contradiction. Adaptive reasoning is the capacity to think logically about the relationships between situations and concepts. In mathematics, this ability allows the learner to make sense of procedures, concepts, and solutions and see that answers to problems are correct because the reasoning is valid.

Sylvan Learning instructors are trained to ask critical questions, and each Sylvan lesson employs a variety of questions to engage the student in the learning process. Sample questions that teachers may ask include:

- How did you solve this problem?
- What are some different ways we could solve this problem?
- Does this answer seem reasonable? Why or Why not?
- Why do you think this makes sense?
- Show evidence to demonstrate that your answer is correct.

Within the Sylvan mathmatics lessons, students have opportunities to develop their reasoning skills through problem-solving activities that require logical and critical thinking. This Applied Practice Activity asks students to select which of two solutions is correct, or to determine why a solution is incorrect and then to provide the correct solution.

Constructing Viable Arguments (Communication). Although mathematics is often taught as a collection of separate strands, it is an integrated field of study. When teachers make connections between mathematical ideas, students begin to understand how these ideas build on one another. For example, students are first taught to add. Later, multiplication can be seen as repeated addition. Instead of seeing mathematics as a set of arbitrary rules, students should build on previous knowledge and experiences.

Students who have a conceptual understanding grasp more than isolated facts and methods. They have organized their knowledge into a coherent whole that enables them to learn new ideas by connecting those ideas to what they already know. They have less to learn because they see the similarities between seemingly unrelated situations. Procedural fluency goes hand in hand with conceptual understanding and includes knowledge of procedures, knowledge of when to use them appropriately, and the skill to perform them flexibly, accurately, and efficiently.

In this lesson, students relate multiplication and division facts.

Follow this link to a sample lesson in which students connect fractions and decimals.

In this lesson, students combine skills from coordinate geometry and algebra to solve real-world problems.

Sylvan Learning's mathematics programs help to develop the communication skills needed to speak, read, write, and listen mathematically. Central to Sylvan Learning's lessons is the direct instruction of mathematics vocabulary as it is introduced. Vocabulary is reinforced throughout the lesson discussions and in subsequent lessons. Students are encouraged to use correct mathematical names and terms when they ask questions, discuss solutions, write explanations of their solutions, and summarize their understanding in Learning Logs.

This lesson illustrates how vocabulary is introduced in the Guided Practice.

Communication. Because mathematics is often conveyed as symbols, being able to communicate about it may not be seen as important, and students may find it difficult to talk about mathematics. However, they must be able to speak, read, write, and listen mathematically so that they can organize, present, and justify their reasoning clearly. Students who can do so are able to analyze and evaluate the mathematical thinking and strategies of others and express mathematical ideas with precision. Communication skills are developed through the grades: At the elementary level, students may draw a picture instead of writing, but by the time they reach middle and high school, they should be able to present mathematical ideas in a more rigorous fashion. General communication skills are developed as part of the literacy program, but teachers should provide opportunities for communication in mathematics.

Modeling with Mathematics (Representation). The ways in which we understand and use mathematical ideas depend on the ways in which those ideas are represented. A simple illustration is how we represent numbers using the base-ten form as a way to understand regrouping for addition and subtraction. In *Principles and Standards for School Mathematics* (PSSM), representation refers to both process (the act of capturing a mathematical concept in some form) and product (the form itself). Students should be able to create and use representations to organize, record, and communicate mathematical ideas. They should also be able to use conventional forms of representation. As students progress through the grades, they should be able to select, apply, and translate among mathematical representations to solve problems. This might mean choosing to use algebraic expressions, deciding whether a bar or circle graph is appropriate to illustrate a problem, or using the computer to generate a spreadsheet. The term "mathematical model" can refer to the use of physical manipulatives or to drawing a geometric model. Representations that model and interpret physical, social, and mathematical phenomena are tools that aid students in understanding and communicating mathematics.

Sylvan Learning's mathematics programs support students' ongoing development of representation through the use of many forms of mathematical models. New concepts are typically introduced using manipulatives, and students are encouraged to move through the developmental stages: from concrete to pictorial to symbolic to abstract representation of concepts. Manipulatives are provided as necessary to support and reinforce the learning process and movement through the various stages.

Follow this link to see a sample lesson in which students model, write, and complete addition and subtraction number sentences.

In this lesson, the concept of ratio is introduced with pictorial representations.

Click here to see a lesson in which students use a variety of representations to describe functions.

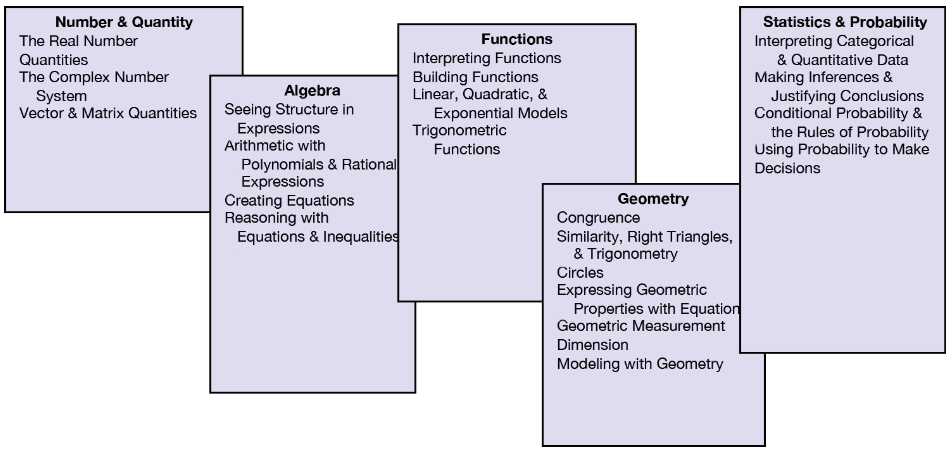
Using Appropriate Tools Strategically. The new process standards describe the range of tools now available to students-traditional tools such as pencil, paper, and rulers, but also calculators, statistical packages, digital content, and increasingly dynamic, interactive computer software-as well as the importance of deciding what and when certain tools are appropriate. These tools should help students represent and analyze mathematical relationships, thus deepening their understanding rather than doing the thinking for students.

Attending to Precision. The new standards also emphasize the importance of calculating, notating, and explaining mathematical solutions with precision appropriate to the grade level. Younger students are encouraged to explain to teachers and peers how they've come up with solutions; as students proceed through the grades, they develop more precision, learning how to defend their claims and use definitions to explain them.

Looking for and Making Use of Structure and Repeated Reasoning. Part of thinking mathematically is being able to see patterns and understand the structure of problems and relationships. Younger students notice shapes and quantities; as they get older, they translate these into operations, properties, and geometric figures and proofs. As students become more mathematically proficient, they begin to see not only patterns but repetitions of patterns, such as how terms cancel each other out, or how decimals repeat. Noticing these patterns helps them to develop shortcuts and general methods or formulas for solving problems, always, as in other practices, checking to see if their methods make sense.

Standards For Mathematical Content

The Common Core Standards for Mathematical Content are organized by conceptual categories that define the understanding, knowledge, and skills that children should acquire from grades K through 12. The K–8 standards include Counting and Cardinality (kindergarten), Operations and Algebraic Thinking (K–5), Number and Operations in Base-Ten (K–5), Number and Operations–Fractions (3–5), Measurement and Data (K–5), Geometry (K–8), Ratios and Proportional Relationships (6–7), the Number System (6–8), Expressions and Equations (6–8), Functions (8), and Statistics and Probability (6–8). Each standard contains ideas that are integral to all the grades, but the depth to which each is addressed is appropriate to the noted grade level. The high school standards include the five domains in the graphic below. The new standards also include Modeling as a domain, but one that is woven throughout the others rather than having its own unique set of topics.



Number and Quantity. This standard forms the basis for all of the other standards. Instructional programs at all grade levels should enable students to understand numbers, ways of representing them, relationships between them, and number systems (Fuson, 2003). This begins with students learning to count with understanding and connecting words and numerals to the quantities they represent. They begin with whole numbers and continue to develop number sense with fractions, decimals, percents, and integers (Fuson, 2003). Students should understand the meanings of the operations and their inverses and how they relate to one another, as well as the associative, commutative, and distributive properties. They should compute fluently with whole numbers, fractions, and decimals and make reasonable estimates. Students should be able to use visual models and choose appropriate methods to solve problems.

Some students, particularly students whose primary language is not English, may struggle with numbers and operations, such as naming large numbers, since naming conventions are not uniform across units (tens, hundreds, thousands). Many textbooks "show little understanding of children's progression of methods" (Fuson, 2003, p. 74). Studies have shown that errors in mathematical computations and fluency are hard to correct over time because students may revert to old habits even after realizing their mistakes (Resnick & Omanson, 1987).

Follow this link to see a lesson in which students practice subtraction with regrouping.

In this sample lesson, students check solutions to division problems using multiplication.

In this lesson, students learn the properties of multiplication and use them to solve word problems.

Click here for a lesson in which students practice their estimation skills to develop number sense with fractions.

Algebra. As noted previously, a significant amount of attention has been given to the study of the teaching of algebra, and many mathematics educators now call for earlier introduction of algebraic thinking into the curriculum. This shift in focus has produced what Kaput (2007) has labeled "the algebra problem": "the highly dysfunctional result of the computational approach to school arithmetic and an accompanying isolated and superficial approach to algebra [that has] led to both teacher alienation and high student failure and dropout, especially among economically and socially less advantaged populations" (p. 6). The importance of algebra is well documented in the research on its longer-term effects: Students who complete Algebra II graduate from college at more than double the rate of those who do not.

Algebra standards in grades K through 8 should enable all students to understand patterns, relations, and functions (Kaput, 1999). This process begins with working with patterns in the lower grades, as students recognize, generate, and analyze repeating patterns. As they move into higher grades, they describe and extend patterns and begin to make generalizations with graphs, words, and symbols. Students should represent and analyze mathematical situations and structures using algebraic symbols. They should understand the idea and use of the variable and use symbols to represent situations and solve problems. The use of mathematical models such as graphs, tables, and equations to represent and understand quantitative relationships is important. In the upper grades, students should be able to analyze qualitative and quantitative change in various contexts, use equations, and understand functions.

Kaput (1999) notes that during this learning process, "different aspect of algebra become habits of mind, ways of seeing and acting mathematically-in particular, ways of generalizing, abstracting and formalizing across the mathematics and science curricula, including curricula leading to the world of work" (p. 135). This becomes important when one considers that research has shown that many students experience serious issues in their ability to solve problems (Brown, Carpenter, Kouba, Lindquist & Silver, 1988; Dossey, Mullis, Lindquist & Chambers, 1988; Travers & McKnight, 1985).

Click here to see a lesson that introduces students to algebraic thinking with skip counting and pattern identification.

Follow this link to see a sample lesson in which students begin to analyze more complex picture and number patterns.

Functions. The new CCSSM introduce functions in eighth grade, when many students are already familiar with or have an intuitive understanding of functional relationships-when, for example, they figure out how much four apples cost if they know the cost of one, or how long it would take to drive a certain distance if they know the distance and the speed in miles per hour. The early introduction of algebraic thinking also accustoms students to expressing functions as simple equations, or a set of inputs and outputs, and solving for the unknown value. The other concept that students learn early in their instruction about functions is how to test whether a relationship is a function: that is, functions have only one output for a given input.

The high school standards deepen students' understanding of functions and the relationships between quantities. Students also learn how to describe functions in different ways-by graphs, a written or verbal rule or formula, a table, or a recursive rule. They also learn how to interpret or analyze functions using these different representations, mapping one set of inputs to a set of outputs or vice versa. As part of their algebra courses, students are introduced to ordered pairs and the notions of domain and range. They gradually begin to build functions that model relationships and begin to understand and compare linear, quadratic, and exponential models. As they get into higher-level mathematics, they learn notation and trigonometric functions.

Follow this link to a lesson in which students work with ordered pairs and graphing functions.

In this lesson, students learn to use function notation and compute the value of a linear function for given values of the independent variable.

This lesson introduces the concept of translating quadratic functions.

Geometry. In the U.S., geometry instruction has traditionally included an informal introduction to a few basic concepts in grades pre-K through 8-shapes, their properties, spatial relationships-and then focused on an axiomatic, Euclidean geometry in high school. Recent research and international practice, however, show that much more "can and should be done in all grades" (Clements, 2003, p. 151). This conclusion is based on findings from over a decade of research showing that U.S. students underachieve in geometry when compared to their international counterparts (Beaton et al., 1996; Lappan, 1999; Stigler, Lee, & Stevenson, 1990), and that students' geometry content is not connected from grade to grade (Clements, 2003; Mullis et al., 1997).

The NCTM Standards set the stage for a major reemphasis on geometry education for all students grades pre-K through 12 (Clements, 2003). According to the standards, geometry instruction should enable K-8 students to analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships. Students should be familiar with the properties of congruence and similarity and be able to specify locations and describe spatial relationships using coordinate geometry and other representational systems. Students should be able to apply transformations (rotations, reflections, translations, dilations) and use symmetry (line and rotational) to analyze mathematical situations. They should know how to use visualization, spatial reasoning, and geometric modeling to solve problems not only in geometry but also in other areas such as number, measurement, and algebra.

More broadly, the study of geometry should help students make connections among the various topics of mathematics and apply their knowledge to real-world situations. Properties of congruence and similarity, for example, provide opportunities for students to practice proportional reasoning skills that help strengthen their understanding of fractions, decimals, and percents. Transformation and symmetry help prepare students to graph equations in the coordinate plane but also to analyze the basic structure of equations and their graphs.

The Van Heile model is integral to how geometry *should* be taught. It breaks geometry lessons into five levels of thought and has several basic assumptions: First, "learning is a discontinuous process characterized by qualitatively different levels of thinking"; second, "[the] levels are sequential, invariant, and hierarchical"; third, "concepts implicitly understood at one level become explicitly understood at the next level"; and fourth, "each level has its own language and way of thinking" (Clements, 2003, p. 152).

According to Clements (2003), the five levels of thought are formalized as follows:

- **Level 0: Children do not reliably distinguish shapes and are unable to form reliable mental images of these shapes.**
- **Level 1 (Visual Level): Students can recognize shapes as wholes. Properties of shapes are not considered.**
- **Level 2 (Descriptive/Analytic Level): Students recognize and characterize shapes by their properties.**

- **Level 3 (Abstract/Relational Level): Students can form abstract definitions and provide logical arguments in the geometric domain.**
- **Level 4 (Rigor): Students can establish theorems within an axiomatic system.**

In this lesson, students are introduced to the concept of congruency by making visual comparison of shapes.

In this lesson, students learn about reflectional and rotational symmetry.

In this lesson, students define, identify, and use geometric transformation.

Statistics and Probability (Data and Probability). Instructional programs should enable students to formulate questions that can be addressed with data and to collect, organize, and display relevant data to answer those questions, using tables, line plots, and picture, bar, and line graphs. They should also be able to use appropriate statistical methods such as histograms, stem-and-leaf plots, box plots, and scatterplots to analyze data and develop inferences and predictions. Beginning in grade 3, students should understand and be able to apply basic concepts of probability, such as describing events that are likely or unlikely, predicting outcomes, and testing conjectures about the results of experiments. By grade 8, students should be able to compute probabilities using methods such as tree diagrams and area models.

Developing a conceptual understanding of statistics, probability, and basic mathematics is important in part because from 1986 to 1996 the percentage of test items classified as probability and statistics rose from 6% to 20% for grade 12 students, and to 15% for eighth graders (Shaughnessy & Zawojewski, 1999).

Probability also plays a role in everyday life. While “bottom line” probability, or hunches, can be a teaching tool for younger students (Davis & Hersh, 1991; Metz, 1988), mathematics educators must provide students with objective ways of approaching data and chance (Shaughnessy, 2003).

Follow this link to a sample lesson in which students are introduced to the concept of probability to determine whether a game is fair or unfair.

In this lesson, students use tree diagrams to list all possible outcomes of an event.

This activity shows students creating and interpreting stem-and-leaf plots.

In this lesson, students learn to calculate the probability of independent and dependent events.

Modeling

Understanding functions is, in part, about modeling relationships, but mathematical modeling includes statistical models, geometric models, or any other models that describe the relationships between variables and help students visualize and analyze mathematical concepts, and relate them to their everyday lives. Thus the CCSSM include modeling not only as one of the content domains but also as a process standard, or mathematical practice that spans multiple content standards and that students should eventually develop as a mathematical habit of mind.

Younger students may write a simple equation, draw a picture, or create a diagram. As they get older they begin to use proportional reasoning, beginning their work on modeling by graphing or plotting points; in higher-level algebra courses, they learn to graph different kinds of functions, either as a traditional graph-often using a graphing calculator-or in tables, formulas, or other representations. Modeling is a way of understanding relationships, a kind of problem solving in which students identify the variables, come up with a model, perform a set of operations to draw conclusions, consider the results, and validate the conclusions to see if they hold true when different values or inputs are substituted. According to the CCSSM, one of the "insights" provided by mathematical modeling is that the same mathematical or statistical structure can "sometimes model seemingly different situations" (CCSSM, 2010)

Advanced Mathematics

Work on the new standards used the structure of mathematics and educational research to map learning progressions of topics across grade levels. This map was split into grade-level standards, then revised and edited into the CCSSM released in 2010 (Common Core Standards Writing Team, 2013). In addition to more focused content, writers of the CCSSM suggested a shift in the curriculum and in student pathways to include a fourth year of math in high school. Studies show that if students enroll-and have the preparation and skills to succeed-in more advanced math courses such as precalculus, probability and statistics, calculus, modeling, and discrete mathematics, they are more likely to be successful in college-level courses. Research by ACT found that of students taking Algebra I, Geometry, and Algebra II and no other mathematics courses, only 13 percent met the benchmark for readiness for college algebra. One additional mathematics course greatly increased the likelihood that a student would reach that benchmark, and three-fourths of students taking calculus met the benchmark (National Governors Association et al., 2010).

In the Appendix to the new standards, the authors of the CCSSM lay out four "Pathways" or model courses for high school mathematics. In doing so, however, they stress that, "The pathways and courses are models, not mandates.... All college and career ready standards are found in each pathway. The course descriptions delineate the mathematics standards to be covered in a course; they are not prescriptions for curriculum or pedagogy" (CCSS, 2010, Appendix A, pp. 4-5).

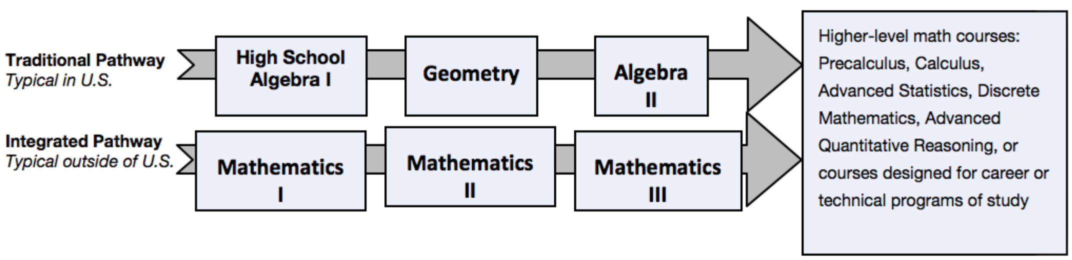
The Pathways

1) An approach typically seen in the U.S. (Traditional) that consists of two algebra courses and a geometry course, with some data, probability, and statistics included in each course

2) An approach more typically seen internationally, but increasingly adopted in the U.S. as well (Integrated) that consists of a sequence of three courses, each of which includes number, algebra, geometry, probability, and statistics

3) "compacted" version of the Traditional pathway where no content is omitted

4) A "compacted" version of the Integrated pathway where no content is omitted



In laying out the alternative pathways, the authors of the CCSSM acknowledge that students progress at different rates and that many need extra support, provided through extended class time, special classes, or after-school tutoring (CCSS, 2010, Appendix A, p. 5). Citing research that shows that "allowing low-achieving students to take low-level courses is not a recipe for academic success" (Kifer, 1993), they also emphasize the importance of making available both support and a range of high-quality mathematical courses so that all students can complete their high school career with a deeper understanding of the standards (CCSS, 2010).

Research on the New Standards. The implementation of the CCSSM is in its infancy, so it is too early for conclusive findings, but there are studies that look ahead to the impact of the CCSSM on math performance. The first is a study that looked at how the standards themselves measure up to the international standards that seem to make students more academically competitive. Researchers at the University of Michigan used correlational analysis, regression, and analysis of covariance techniques to compare the CCSSM with the standards of nations whose students posted the highest achievement on the TIMSS. Looking at state standards in place in 2009, they assessed the relationship between the proximity of a state's standards to the CCSSM and students' performance on the National Assessment of Educational Progress. Researchers found a very high degree of similarity between the CCSSM and the standards of the highest achieving nations on the 1995 TIMSS; they also found that states with standards more like the CCSSM had higher scores on the NAEP (Houang & Schmidt, 2012).

A second study, referred to earlier and conducted by ACT in 2010, took a "first look" at the Common Core and college and career readiness, or current achievement levels relative to the new standards. Researchers examined the performance of 256,765 eleventh grade students who took the ACT as part of their state's annual testing, comparing their scores for clusters of skills to ACT's College Readiness Benchmarks. The results indicated that educators implementing the CCSSM and preparing students for the new assessments, as well as for college and careers, have their work cut out for them: only about a third to a half of the students achieved the desired level of readiness (ACT, Inc., 2010).

There is also a growing number of CCSSM papers and educational resources to help educators prepare students. The professional journal *Teaching Children Mathematics* (published by the NCTM), for example, introduced a series of five articles that examined the major ideas contained in the math standards and addressed strategies for specific grade bands. The National Council of Supervisors of Mathematics, through a sponsorship by Carnegie Learning, is currently developing materials and resources and organizing conferences, podcasts, and other initiatives to promote and share CCSSM resources (Mitchell & Schrock, 2013).

HIGH QUALITY ASSESSMENT

A trend that in many ways runs parallel to the development of the new standards is the growing emphasis on high-quality assessment: To ensure that students are fully prepared and have mastered the skills needed to succeed on high-stakes assessments, teachers need to determine what students know and are able to do at each step along the way. In a 1995 document, the NCTM summarized four major purposes for assessment: (a) evaluating student achievement, (b) evaluating programs, (c) monitoring student progress, and (d) making instructional decisions. While all are still relevant, recent focus has been on the latter two. According to the 2000 *PSSM*, mathematics assessment should support the learning of important mathematics and furnish useful information to both teachers and students.

Assessments should reflect the mathematics that students should know and be able to use; enhance students' mathematics learning; be an open and coherent process; and promote valid inference. Most importantly, assessment should be an integral part of ongoing classroom activities and provide useful information to teachers about what students are learning so that they can support student progress. The key strategies for effective assessment are:

- Clarifying and sharing learning intentions and criteria for success
- Engineering effective classroom discussions, questions, and learning tasks that elicit evidence of learning
- Providing feedback that moves learners forward (Lester, 2007, p. 1054)

In recent years, formative assessments have garnered increasing attention because they not only help teachers make decisions about content and instructional strategies, but they also provide students with useful feedback about how they are doing and how they can improve. The learning of students, including low achievers, is generally enhanced when teachers use formative assessment in making judgments about teaching and learning (Black & Wiliam, 1998). Ongoing formative assessments can be as simple as the teacher questioning a student or observing how a student completes a task, or they can involve more complex learning cycles in which teachers use performance assessments and data to make instructional decisions. Examples of this cycle could include figuring out when to revisit a concept or adapt instruction for those who are struggling. The critical aspect of formative assessment is that it is not a single tool or instrument, but a process (Wiliam, 2011).

A number of researchers have shown that assessment cycles designed for planning and individualizing instruction are effective in helping students improve academically. Recent research has indicated that periodic assessments used to identify student needs and to adjust instruction can improve student outcomes. For example, the What Works Clearinghouse report on assessment concluded that using an assessment cycle to assess, teach, and adjust instruction is an effective strategy (Hamilton et al., 2009). Similarly, the Institute of Education Sciences, in its guide for Response to Intervention (RTI), emphasizes the importance of assessment at the beginning and middle of the semester, as well as at least monthly for under-achieving children (Gersten et al., 2008).

Sylvan Learning mathematics programs incorporate external formative assessments to determine where each student should start; these assessments determine an approximate place on a learning progression and monitor student progress against national achievement. Sylvan programs also include embedded formative assessments that are used by teachers to inform instruction along the way. For SylvanSync programs, Sylvan uses the Renaissance Learning's STAR assessments to provide placement-level data and benchmark achievement and to create individualized plans. Following the initial assessment, teachers monitor ongoing skill and knowledge development that has occurred as the result of Sylvan Learning programs. Instructional modifications are made based on students' daily performance and performance on these assessments. Progress assessments are administered at 24-session intervals using the STAR assessments to measure growth, guide future instruction, and evaluate students' overall progress.

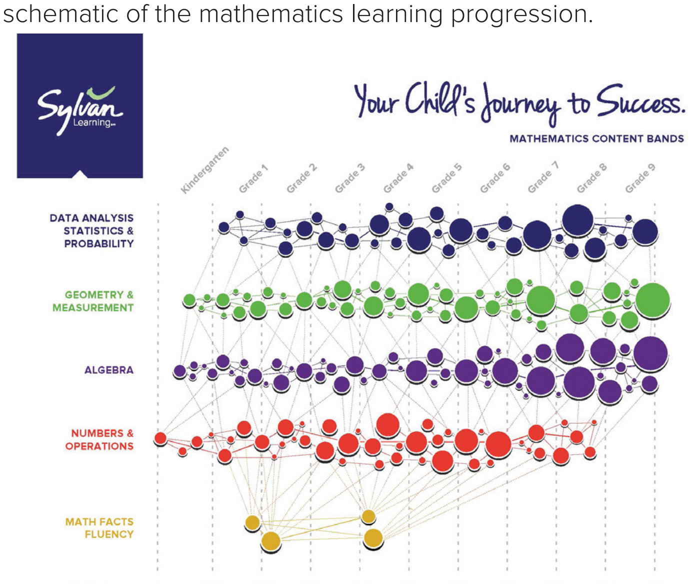
Follow this link to an illustration of a learning object. Students who do not demonstrate mastery on the pretest for a skill will progress through a series of learning objects including guided practice, independent practice, applied practice, and mastery tests. At each step students are assessed and progress to the next learning object only if they have demonstrated understanding at the appropriate level. Students who do not immediately show understanding are given additional content instruction and support from the teacher until they are ready to move on. When appropriate, additional prerequisite skills may be added to the student's learning plan. This link will take you to a sample learning plan.

ADAPTIVE LEARNING AND LEARNING PROGRESSIONS

With advances in computer technology, the popularity of adaptive computer programs has risen in the last decade. Adaptive tests tailor questions to students' ability levels, posing more difficult questions following correct answers and easier questions following incorrect answers. Similarly, adaptive learning programs deliver personalized instruction depending on student needs and ability level. A study conducted by the Parthenon Group (2011) found that personalized learning is faster, often accelerated learning, and thus ideal for students who have fallen behind.

Sylvan Learning's digital teaching platform, SylvanSync, includes new mathematics content, which is mapped to the CCSSM. The most recent version of SylvanSync also employs Sylvan's new adaptive backbone, which is based on an empirically valid learning progression. Sylvan created this progression by mapping the new content to a learning progression originally developed by Renaissance Learning. These progressions were based on the analysis of data from millions of students who took STAR tests as benchmark tests of their progress toward state test proficiency-and by carefully studying the most logistical path that students take in acquiring competency in reading and mathematics. Adaptive curriculum and assessments provide the motivation, engagement, and individualization students need to be successful. "The SylvanSync platform helps track student progress and identifies the most appropriate learning resources for each student," removing much of the administrative burden associated with more personalized approaches to instruction (Richards & Dede, 2012). With the help of SylvanSync, students begin instruction at the appropriate level, progressing at an individualized rate. Sylvan tutors are free to focus their attention on student interactions and remediation techniques.

Below is the schematic of the mathematics learning progression.



TECHNOLOGY AND LEARNING MATHEMATICS

Several overviews of the effects of using instructional technology, conducted in the 1990s, demonstrated its usefulness in teaching and learning mathematics. In particular, computer use was associated with increased performance when students had adequate access to up-to-date computer technology and when computers were used to help students learn higher-order concepts. In a review of research on three categories of computer-based mathematics learning-programming, computer-assisted instruction, and mathematics tools-McCoy (1996) found several studies that showed that knowledge of programming increased elementary students' understanding of geometry, and that it appeared to have a positive effect on mathematical problem solving at various age and grade levels. These studies also showed that the use of math tools that assist with math functions, such as graphing and symbolic calculators, could lead to significantly higher achievement in conceptual areas and in computation and manipulation skills. A meta-analysis conducted in the 1990s showed positive effects on mathematics achievement for Integrated Learning Systems (Kulik, 1994) and specific technology-based mathematics programs, such as *The Adventures of Jasper Woodbury* (Hickey, Moore, & Pellegrino, 2001). At the secondary level, Koedinger, Anderson, Hadley, and Mark (1997), studying the effectiveness of the Pittsburgh Urban Mathematics Project (PUMP)'s *Cognitive Tutor Algebra* on ninth-grade students' algebra performance, found that, across teachers, students performed better with *Cognitive Tutor*.

Technology use, access, and options have all increased dramatically since these early studies. Computer-based or assisted instruction now involves far more sophisticated and interactive tools and assessment capabilities. Students and teachers now use everything from internet resources, to game play, to mobile technologies as part of their math activities. It is fairly commonplace for major reform efforts and documents like the CCSSM to encourage the "strategic use of technology" and recommend using technology tools to help students test their ideas or mathematical solutions, create graphs and visualize data, or come up with geometric constructions (CCSSM, 2010, Appendix A, p. 4). The guidelines also recommend incorporating technology into testing. The use of technology has also been supported by national organizations (PSSM, 2000) and by recent research.

Most researchers and practitioners exploring the value added by technologies in mathematics teaching and learning agree that technologies do not replace the need for computational fluency and competence with standard algorithms for addition, subtraction, multiplication, and division. Nor do they replace the teacher (Heid & Blume, 2008). What technologies do provide are new levels of richness in interaction with content and opportunities to represent a variety of concepts in novel and effective ways. Both of these factors contribute to technologies' capacity for powerfully engaging learning. Dick (2008) notes that "students learn mathematics by taking mathematical actions…on mathematical objects…observing the mathematical consequences of those actions, and reflecting on their meanings" (p. 334). These are actions that technologies can enhance as students interact with or create visual representations of mathematical ideas.

The following sections review some of the more recent technology research and tools incorporated by educators who, like Sylvan Learning, are steadily engaged in the process of deciding when and how to take advantage of the wide-ranging possibilities of digital technologies.

Math Programs and Software

Multimedia. The efficacy of educational multimedia is supported by a variety of learning science research. For example, one quantitative measure of engagement and attentiveness is eye movement. In searching for optimal combinations of animation, text, simulation, and narration, She and Chen (2009) found that greater focus on pertinent material (measured through eye movement) increased post-test performance. Many mathematical concepts have been explored using multimedia applications. In looking at kindergarten students' understanding of fractions, Goodwin (2008) found that the use of multiple media facilitated exceptionally sophisticated fraction understanding, primarily due to three key multimedia affordances: manipulation of virtual artifacts, multiple representations, and immediate feedback.

Although these technology tools are not currently in SylvanSync, Sylvan has plans to develop and integrate multimedia components within the lessons.

Gaming Environments. The educational potential of games is a rich area of research, focusing primarily on how the engaging qualities of commercial video games can be leveraged for educational purposes. Research has demonstrated that while playing commercial video games, students engage in learning that can exceed comparable learning in many traditional educational environments. One of the most compelling cases to be made for the use of games in mathematics education is their obvious strength in motivation and engagement. Gee (2004, 2008), who has studied the learning that takes place during game play and worked to understand how games facilitate learning, identifies three learning principles that appear to various degrees in most compelling commercial games: empowerment (based on the satisfaction of developing one's own identity and customizing one's own world), problem solving (pushing against the "pleasantly frustrating" and engaging edge of challenge), and understanding (based on comprehending how a game world works) (Gee, 2004). Gee also has examined the learning in games from the perspective of discovery-oriented play, highlighting parallels between how players come to understand the workings of a game world and how learners understand physical laws (2008).

Although the SylvanSync Mathematics program currently does not incorporate a gaming environment, parents are given access to the mySylvan portal where students can access additional activities, including educational games.

Simulations. By allowing learners to interact with content in a contextualized manner, simulations can be powerful educational artifacts, particularly in the areas of science and mathematics. Simulations are frequently used to model real-world phenomena, such as city traffic or competing animal populations in the wild; by setting and resetting initial parameters and observing the simulation as it unfolds, learners can develop an understanding for the dynamics at work. Based on an extensive literature review, Means and colleagues (1993) concluded that simulation programs tend to motivate students, increase productivity, and promote advanced skills and knowledge. They also found that simulation software fosters student-centered learning environments and promotes student/student and student/teacher collaborative learning.

Learners can delve deeper into simulations for potentially greater learning gains. Creating simulations has been shown to increase students' understanding of content, and, because students focus their creations on topics with personal meaning, increase engagement and motivation (Repenning, Ioannidou, & Phillips, 1999). Parush, Hamm, and Shtub (2002) show that students who are given the opportunity to rewind simulations, alter parameters, and increment one step at a time spend more time engaged with the simulation and demonstrate deeper post-test content understanding than those who simply run a simulation from beginning to end. Furthermore, the post-test trend continues after the experimental capabilities are removed.

Sylvan has plans to integrate many of these technologies into the release of future SylvanSync Mathematics programs.

Tablet Computers. A number of studies point to the motivational aspects of delivering instruction via tablets computers (see, for example, Rockman et al, 2012). Studies are also beginning to emerge on the impact of iPads and other mobile technologies on student learning, and on math learning in particular. A recent experimental study was conducted by researchers at the University of South California, in partnership with GameDesk, a nonprofit organization which originated at USC to create, test, and evaluate effective game- and play-based software curriculum. This study found that fifth graders playing a fractions game for 20 minutes daily on iPads posted higher test scores and more positive attitudes about fractions than the control group (Truta, 2011). A year-long randomized control study with middle schoolers comparing the use of the Houghton Mifflin Harcourt algebra textbook to the *HMH*: Fuse iPad app of the book, showed that 78% of eighth graders using the iPad app scored “proficient” or better on the district algebra exam, versus 59% of those who used only textbooks (Riverside County Office of Education, 2013).

MATHEMATICS INSTRUCTION AND EQUITY

The “Equity Principle” of *Principles and Standards for Mathematics* states that all students can learn mathematics and that there should be high expectations, worthwhile opportunities, and strong support for all students (Gutierrez, 2002). This vision of equity challenges a belief in this country that only some students are capable of learning high-level mathematics. These may not include certain ethnic groups, females, speakers of English as a second language, and those from lower socioeconomic groups. Even with reforms and the development of better mathematics programs to address these issues, unequal access to high-level math curricula and instruction remains. Results of national and international studies, such as TIMSS, have shown that socioeconomic status correlates with performance. Disproportionate numbers of poor, African-American, Latino, and Native American students perform below proficiency levels on tests of mathematical competency and drop out of mathematics, which means that these students are denied both important skills and a critical pathway to financial and career success (Schoenfeld, 2002). NAEP results (Perie & Moran, 2005) have also shown that although the differences in achievement between boys and girls are not as pronounced as they once were, boys continue to have more positive attitudes towards mathematics, and girls’ attitudes towards mathematics decline more sharply through the grades.

Background Differences and Communication

Researchers have found that, even in mathematics, relating an idea to previously acquired knowledge helps students learn. This means that educators and curriculum developers should acknowledge cultural differences in the form of prior knowledge that students bring to mathematical learning (Carey, Fennema, Carpenter, & Franke, 1995). Communication is another key component in learning mathematics. Because many schools serve students whose primary language is not English, it is important to provide additional support so that those students can benefit from communication-rich mathematics classes.

While mathematics is often depicted as a universal language of symbols, it has its own vocabulary, syntax, and format, which present difficulties even for some English-speaking children. Mathematics teachers can show sensitivity to the English language learner by systematically teaching mathematics vocabulary, eliminating the use of idioms, using culturally-relevant problems and illustrations, and incorporating activities that teach reading and writing skills in a mathematical context.

Within the Sylvan Learning mathematics lessons, students are provided with opportunities to develop their language skills through introductory activities that access prior knowledge and through the multiple problem-solving activities that accompany lessons. Students are asked to verbalize their understanding of concepts through verbal and written explanations for the problems they solve and at the end of sessions, when they add comments to their Learning Log.

Follow this link to a sample activity in which students are asked to explain how they estimate a sum and explain if the estimate will be more or less than the actual sum.

This activity asks students to explain how to evaluate an algebraic expression as well as to provide an example.

In this activity, students explain predictions using inductive and deductive reasoning.

Family Involvement

Families of English language learners (ELLs) may also have different perspectives on schooling and varying degrees of understanding of the U.S. educational system (Goldenberg, Gallimore, Reese, & Garnier, 2001). Studies of minority populations have shown that parents want to be involved (Chavkin & Williams, 1993; Delgado-Gaitan, 1992) but may have different notions of participation than mainstream parents (Goldenberg & Gallimore, 1995; Trumbull, Rothstein-Fisch, & Hernandez, 2003; Valdés, 1996). Research has also shown that there are no differences in family involvement across either race or socioeconomic status (National Urban League, 2008), but the type of involvement may be different across populations (Delgado-Gaitan, 1992; Goldenberg, Gallimore, & Reese, 2003; Valdés, 1996).

Sylvan Learning programs serve a wide student population and recognize the cultural diversity that exists in the families of its students. Communicating with families is a high priority for Sylvan teachers and Sylvan Learning Center directors, and each center strives to address the needs of its parent population and incorporate parents as partners in their children's educational programs. To encourage initial and ongoing communication with these families, Sylvan's small group programs, informal meetings, and conferences can be scheduled after school and during evening hours that are convenient for families and that may provide a more relaxed atmosphere for parents.

MOTIVATION, LEARNING, & ACHIEVEMENT

Educational psychologists and social cognitive theorists have long explored the role of motivation in student learning and achievement and generally agree that students of all ages need both cognitive skill and motivational will to do well in school (Pintrich & Schunk, 2002). They need what a recent comprehensive review of the relevant literature on the non-cognitive factors that shape students' performance terms an "academic mindset" (Farrington et al., 2012), which includes being engaged in learning, confident in one's own abilities, and willing to persevere at even difficult tasks.

Recognizing the impact of motivation on student achievement, Sylvan Learning has developed a Student Motivation Program that focuses on encouraging and recognizing the positive student behaviors that characterize successful students. The Motivation Program employs a system of positive reinforcement and establishes a positive learning environment that fosters student growth and achievement by providing multiple opportunities for individual attention and encouragement. Each program component recognizes individual student accomplishments, reinforces desired behaviors, and creates a positive student-learning environment. The environment also maximizes student-teacher interaction. Both the individual and small-group instructional settings allow for teachers and students to work in close proximity and maintain direct eye contact, which ensures a quick response between exchanges.

Research shows that motivation and a sense of self-efficacy are positively related to higher levels of achievement and persistence on difficult tasks (Bandura, 1997; Eccles, Wigfield, & Schiefele, 1998; Pintrich & De Groot, 1990; Pintrich & Schunk, 2002), and, more specifically, that they can have a positive effect on students' performance in mathematics (Bandura, Barbaranelli, Caprara, & Pastorelli, 1996; Pajares, 2005; Schunk & Cox, 1986).

Because of mathematics' central place in the curriculum and high-stakes testing and the recent emphasis on the math skills required for college and career readiness, researchers are paying increasing attention to the role of motivation in mathematics. What they are finding is that students' sense of self-efficacy correlates strongly with attitudes toward math and problem-solving performance. Students with higher test scores tend to have more positive beliefs about their math skills (House & Telese, 2008); they not only see themselves as good problem solvers but are willing to tackle more complex problems-the kinds of problems likely to appear on the new Common Core-aligned assessments. Conversely, students who do not think that they can solve challenging problems may give up before they try (Hoffman & Schraw, 2009).

The research also suggests that these self-perceptions can be changed. Motivation, most researchers now agree, is not an all-or-nothing characteristic, nor a fixed trait. Students can be motivated in multiple ways and to multiple degrees, depending on the context of their learning and instructional designs and settings, and teachers' efforts can make a difference. The research points to a number of strategies that can foster higher levels of engagement in math-for example, real-life word problems that students can identify with and engage in: dollars for a mini-economy or visual displays of students' learning trajectories and accomplishments. Research has also shown that token systems are effective across various grade levels, school populations, and school behaviors (Kazdin, 1982; McLaughlin & Williams, 1988; O'Leary & Drabman, 1971; O'Leary & O'Leary, 1976; Williams, Williams, & McLaughlin, 1991).

These strategies and forms of support are particularly important for mathematics instruction, because math does not come naturally to all students, and students who have experienced frustration and failure need to re-engage in the learning process and understand that they can succeed. Zimmerman (2000) found evidence that "self-efficacious students participate more readily, work harder, persist longer, and have fewer adverse emotional reactions when they encounter difficulties than do those who doubt their capabilities" (p. 86).

In 2011, Sylvan Learning launched a study of how the Sylvan experience, and particularly the new SylvanSync digital teaching platform, might improves student engagement and motivation. A key part of the study, based on research on students' attitudes toward learning, was the development of a Student Outlook survey, which helps track changes in students' attitudes as they progress through a Sylvan program, and enables Sylvan to explore key links between attitudes and achievement (Rockman et al, 2013).

FEATURES OF HIGH QUALITY INSTRUCTIONAL DESIGN

Mastery Learning

Mastery learning, first described by Benjamin Bloom (1968) and refined and modified by others (Block, 1971; Block & Anderson, 1975; Keller, 1968), involves establishing a performance level identified as "mastery," regularly assessing student progress, and providing corrective instruction to enable students to reach mastery on a final assessment. This approach assumes that with the right amount of time and resources, most students can master instructional objectives (Slavin, 1989). Guskey and Pigott (1988), who conducted a meta-analysis of studies, found that "group-based applications of mastery learning yielded consistently positive effects on a broad range of student-learning outcomes, including student achievement, retention, involvement in learning activities, and student affect" (p. 213).

Small-Group Instruction

Small-group instruction emphasizes diversity rather than uniformity. Instructional methods include ability grouping, reciprocal peer tutoring, and unstructured group work (Abrami et al., 1995; Lou et al., 1996). Research shows significantly larger effects of within-class grouping when teachers in the small-group condition had more or different training than those in the whole-class condition; when grouping was based on ability and other factors, such as gender or group cohesiveness; and when teachers used cooperative learning. Research has also shown that effective instruction for children at risk includes more time, repetition, and the intensity best afforded by small-group instruction. Because learning math involves gradually building on skills, it is especially important that teachers identify what skills students lack and give them ample opportunity to practice and master those skills. This often requires the carefully sequenced, individualized instruction and ongoing assessments provided in small-group settings. Students who struggle with math may also need specially designed practice to retain the new skills (Carnine, 1997). Small-group instruction also provides an environment conducive to offering the positive emotional and cognitive support that children at risk need.

Cooperative Learning

Sylvan camp programs and some of the Sylvan remedial programs are delivered in small groups, which allow students more time to discuss and solve problems with peers. Participation is a key factor in learning. The recent focus on "math talk" reflects the emphasis in the new standards for students to conjecture, explain problem-solving strategies, and come up with solutions as part of a group (Cooke & Adams, 1998; Protheroe, 2007). The more students talk together, and the more feedback they receive, the more advances they make in learning. Especially in a classroom with different levels of language proficiency, it is important to encourage everybody to participate.

> Small-group instruction is a core element of some Sylvan Learning programs, with a student-teacher ratio from 6:1 to 10:1. The Sylvan Learning environment facilitates active student engagement and participation. The low student-to-teacher ratio permits teachers to promote participation from all students to solve problems, share ideas, and monitor understanding in a collaborative format.

In this activity, students work in small-group camp programs to review and practice skills with fractions and decimals.

Intervention Instruction

Intervention research shows that children at risk for failure acquire skills more slowly, but still need the same skills as their higher-performing peers in order to develop mathematical proficiency and think mathematically. These children may have a variety of challenges that affect their performance in math-including not only limited fluency and proficiency with math facts, numeracy, and operations, but also limited auxiliary skills, such as a lack of math vocabulary or decoding and comprehension skills, or even test-taking skills (Chard, n.d.; Hong, Sas, & Sas, 2006). Again, given the procedural, sequential nature of mathematics, it is especially important that intervention instruction identify deficits early, build procedural fluency, and prevent future deficits (Clements & Srama, 2007).

The defining characteristics of mastery learning are the establishment of a performance level identified as "mastery," regular assessment of student progress toward that goal, and the provision of corrective instruction to enable students to reach mastery on a subsequent assessment. As discussed above, mastery learning is key to the success of interventions: "What defines mastery learning approaches is the organization of time and resources to ensure that most students are able to master instructional objectives" (Slavin, 1989, p. 99). The approach rejects the idea that differences in student aptitude will determine corresponding differences in performance. Rather, it is assumed that applying the right amount of time and resources can result in mastery for any student level. Supplemental educational services, then, provide the extra time for the corrective instruction that is needed in addition to school-day classroom instruction.

Students who are enrolled in Sylvan Learning programs are given a diagnostic placement assessment to identify their mathematical skill deficits and determine an instructional level that will challenge but not discourage their learning efforts. After initial testing, individual assessment results are used to create individual plans with specific objectives. These objectives are recorded and aligned with the learning objectives of other students for the purpose of grouping. This individualized process helps Sylvan Learning teachers provide lessons that meet the instructional objectives of each of the students. Throughout instruction, students are assessed frequently to refine instructional objectives as needed and to ensure that learning has occurred.

SYLVAN'S INTERVENTION FRAMEWORK AND INSTRUCTIONAL DESIGN

Sylvan Learning's framework draws on Kame'enui's (2004) work on "Levels of Intervention," a three-tiered approach to remedial academic services for students who are not performing at grade level. The classroom teacher provides the Level I core instruction for the school-wide population. What Sylvan provides for students who need additional help is Level II small-group instruction and Level III individualized instruction, both designed to supplement and enhance what children learn in the classroom. (See Table 2.) In a Level II's small-group setting, or in an individualized Level III-type program, Sylvan students have opportunities to reflect on their earlier experiences and make meaning of their new ones: for example, Learning Log prompts that allow students to reflect on their understanding and learning experience at the end of each tutoring session.

Table 2. Sylvan Intervention Framework

Level	Target Population	Program Description	Grouping	Interventionist	Setting	Sylvan Programs
I	School-wide Population	Core Instructional Program	Full Class	Classroom Teacher	Regular Classroom	n/a
II	Students not achieving at grade level & whose learning needs have not been met by Level I	Core instructional program + Sylvan Learning supplemental instruction. More systematic, intensive, & explicit than Level I programs	Homogenous small groups with 6:1–10:1 student-teacher ratio	Highly trained Sylvan Learning Instructor	Outside of the regular classroom, during the day, or after school	Ace It! Mathematics and Fit for Algebra camps
III	Students who demonstrate sustained lack of adequate progress despite intervention activities provided at Levels I & II	Core instructional program together with Sylvan individualized instruction and intervention	Individualized instruction with reduced student-teacher ratio (3:1–1:1)	Highly trained Sylvan Learning Instructor	Inside or outside of the regular classroom, during the day, or after school	SylvanSync Mathematics

SYLVANSYNC LESSON DESIGN FORMAT

Learning Log

At the beginning of each Sylvan Learning session, the teacher provides students with an opportunity to think about thinking in the form of the Learning Log prompt. A teacher could ask, “How will you use what you learned in your session today?”, “What questions do you have about what you learned today?”, or “What ideas do you want to know more about?” At the end of the session students reflect, in writing, about their understanding and learning strategies-how they went about thinking about the topics in the introduction, and how they applied their learning to items in the Try Together and Independent Practice sessions. Deliberate and active, this form of metacognition helps students consolidate their learning.

Below is a sample of a Student Learning Log.

Sylvan Learning

STUDENT LEARNING LOG

Prompts

To conclude each session, ask students to provide a written response to an assigned prompt.

K–2

1. What questions do you have about what you learned today?
2. What is the most important thing you learned today? Why is it important?
3. How will you use what you learned in your session today?
4. Draw a picture showing something you learned in today’s session. Explain how you feel about what you learned.
5. Describe something you are proud of learning and explain why you are proud.

3–5

1. What is the most important thing to remember from your session today? Why do you think it is so important?
2. What did you learn today that will help make you a better student? How will it help?
3. Write an e-mail to a friend, teacher, or family member explaining a new word, idea, or skill that you learned today. Or, describe something you are proud of learning and explain why you are proud.
4. Name something that you learned today that you know will require more practice before you have mastered it. Explain why.
5. What questions do you have about what you learned today, or what ideas do you want to know more about?
6. Draw a picture of what you are proud of learning today. Explain how it relates to what you learned.

6–8

1. Which learning strategies worked well for you in this session? What was challenging for you today? Explain why.
2. How can you use a skill you learned in your session today in another subject, such as math, English, social studies, or science?
3. Name something that you learned today that you know will require more practice before you have mastered it. Explain why.
4. Write an e-mail to a teacher, friend, or family member explaining something that you are proud of learning today and how it will help you in school.
5. List the key words to remember from your session today. Pick one and explain why it is important, or how it relates to school or your daily life.

9–12

1. Summarize in one paragraph what you learned today. Explain how you can use what you learned in school.
2. How can you use a skill you learned in your session today in another content area, such as math, English, social studies, or science?
3. What suggestions would you give to other students working on the concepts you learned today? Why?
4. List three ways you think your understanding of a skill or topic has improved as a result of today’s session.
5. Name something that you learned today that you know will require more practice before you have mastered it. Explain why.

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Lessons or Intended Learning Outcomes (ILOs)

Students need direct instruction to learn the essential concepts, principles, and strategies necessary to do mathematics. In order to provide a framework in which students learn to the best of their abilities, SylvanSync Mathematics uses a combination of Guided Practice (GP), Independent Practice (IP), and Applied Practice (AP) lesson objects designed to help students master specific skills. After sufficient practice with the skill, students progress to Mastery Tests in which they demonstrate their understanding and retention of the concept learned. Each of the lesson objects is explained on the following pages.

Pretest

The aim of the pretest, designed to take no more than 20 minutes, is to determine whether a student needs explicit instruction in a given skill. The student works through five items that test all lesson objectives.

Table 3. Elements of Pretests

Element	Description
Objectives	Each ILO and the lesson objects associated with it provide the student with specific objectives, so students see right away what skills they will gain during the course of the lesson object. Objectives reflect the new Bloom's taxonomy wording and include measurable, observable goals that the student will meet by completing the exercises.
Exercises	The student completes 5 different items that measure a student's mastery of the skill objectives.
Evaluation	The instructor scores each item as correct, incorrect, or not assigned. The calculation of the score as a percentage will be displayed to the instructor. The "not assigned" designation should only be used in rare circumstances-such as when the instructor is confident that the student understands the concept and assigns only the odd questions in the short time remaining in a session.

Guided Practice

The aim of the Guided Practice lesson objects is to scaffold instruction for students. The instructor guides the student through each step of skill-learning to ensure that the student grasps the concepts and can perform the skill independently. Designed to take 10 to 15 minutes, the Guided Practice lesson objects are efficient and interactive.

Table 4. Elements of Guided Practice Lesson Objects

Element	Description
Objectives	Each ILO and the lesson objects associated with it provide the student with specific objectives, so students see right away what skills they will gain during the course of the lesson object. Objectives reflect the new Bloom's taxonomy wording and include measurable, observable goals that the student will meet by completing the exercises.
Introduction	The Introduction to the lesson object explains the skill, defines key terms, and links the skill to real-world examples. It is vital that the student understands why the skill is important and how it applies to his or her world.
Examples	The Examples section of each Guided Practice provides sample problems and solutions using a "reveal" feature to explain how the correct answer was reached. Students can work through the sample problems to understand the processes involved in each skill.
Try Together	The Try Together section is an interactive section of the lesson object, designed to give students practice applying their new knowledge of a skill to specific examples. Different question types are presented, and the student works with the instructor to make sure that he or she understands the concepts.
Evaluation	Because the Guided Practice is highly interactive, it can be difficult to place a numeric score on the process. For that reason, Sylvan uses the following scoring process: Excellent (really has it; ready for independent work);Good (strong performance; independent work should be monitored) ;OK (getting there but struggled some; needs additional instruction or a prerequisite skill).

Independent Practice

Once a student has satisfactorily completed the Guided Practice, he or she will move on to Independent Practice. In these lesson objects, the student will demonstrate the ability to independently answer questions that address the target skill. Designed to take 10 to 15 minutes, the Independent Practice lesson objects contain the following:

Table 5. Elements of Independent Practice Lesson Objects

Element	Description
Objectives	Each lesson object provides the student with specific objectives. The student sees right away what skills he or she will gain during the course of the lesson object. Objectives reflect the new Bloom's taxonomy wording and include measurable, observable goals that the student will meet by completing the exercises in the lesson object.
Introduction	The Introduction to each Independent Practice lesson object gives the student a brief reminder of what he or she learned in the Guided Practice.
Exercise	The Exercise section of the Independent Practice presents different question types for students to answer. Each question type is preceded by specific directions to the student. There may be multiple-choice questions, short-answer questions, matching questions, essay questions, or tables and graphs to complete. By completing multiple activities within each learning object, the student can apply different strategies to meet each stated objective.
Evaluation	The instructor scores each item as correct, incorrect, or not assigned. The calculation of the score as a percentage will be displayed to the instructor. The "not assigned" designation should only be used in rare circumstances-such as when the instructor is confident that the student understands the concept and assigns only the odd questions in the short time remaining in a session.

Applied Practice

Once a student has satisfactorily completed the Independent Practice, he or she will move on to Applied Practice. In these lesson objects, the student will demonstrate the ability to independently apply the target skill to solve a variety of problems. Designed to take 10 to 15 minutes, the Applied Practice lesson objects contain the following:

Table 6. Elements of Applied Practice Lesson Objects

Element	Description
Objectives	Each lesson object provides the student with specific objectives. The student sees right away what skills he or she will gain during the course of the lesson object. Objectives reflect the new Bloom's taxonomy wording and include measurable, observable goals that the student will meet by completing the exercises in the lesson object.
Exercise	The Exercise section of the Applied Practice presents different question types for students to answer. Each question type is preceded by specific directions to the student. There may be multiple-choice questions, short-answer questions, matching questions, essay questions, or tables and graphs to complete. By completing multiple activities within each learning object, the student can apply different strategies to meet each stated objective.
Evaluation	The instructor scores each item as correct, incorrect, or not assigned. The calculation of the score as a percentage will be displayed to the instructor. The "not assigned" designation should only be used in rare circumstances-such as when the instructor is confident that the student understands the concept and assigns only the odd questions in the short time remaining in a session.

Mastery Test

Once a student has satisfactorily completed the Applied Practice, he or she will move on to Mastery Test (during the next calendar day of instruction). In these lesson objects, the student will demonstrate the ability to independently apply the target skill to solve a variety of problems. Designed to take 10 to 15 minutes, the Mastery Test lesson objects contain the following:

Table 7. Elements of Mastery Test Lesson Objects

Element	Description
Objectives	Each lesson object provides the student with specific objectives. The student sees right away what skills he or she will gain during the course of the lesson object. Objectives reflect the new Bloom's taxonomy wording and include measurable, observable goals that the student will meet by completing the exercises in the lesson object.
Exercise	The Exercise section of the Mastery Test presents different question types for students to answer. Each question type is preceded by specific directions to the student. There may be multiple-choice questions, short-answer questions, matching questions, essay questions, or tables and graphs to complete. By completing multiple activities within each learning object, the student can apply different strategies to meet each stated objective.
Evaluation	The instructor scores each item as correct, incorrect, or not assigned. The calculation of the score as a percentage will be displayed to the instructor. The "not assigned" designation should only be used in rare circumstances-as when the instructor is confident that the student understands the concept and assigns only the odd questions in the short time remaining in a session.

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